MCH2011 STATICS & STRENGTH OF MATERIALS

Force Systems

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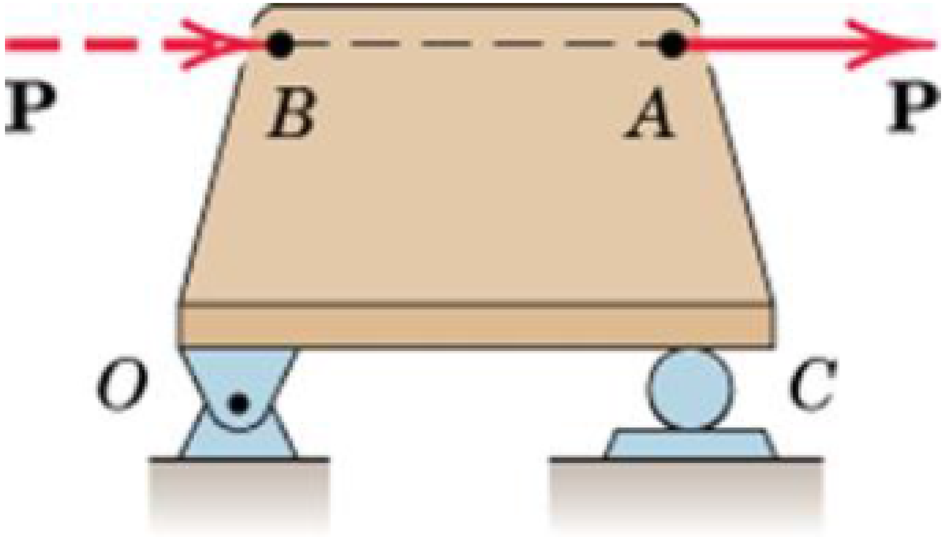
Force Systems – Lecture Objectives

- We will form a vectorial framework for the analysis of a system of forces acting on a body
- With this lecture, we will
  - Learn about sliding (moment) and free (couple) vectors
  - Represent forces , moments, couples, and resultants in 2-D and 3-D
  - Understand the difference between a moment and couple
  - Understand the relationship between forces, moments, and couples
- The concepts introduced in this lecture is the key to understanding static and dynamic analysis of bodies

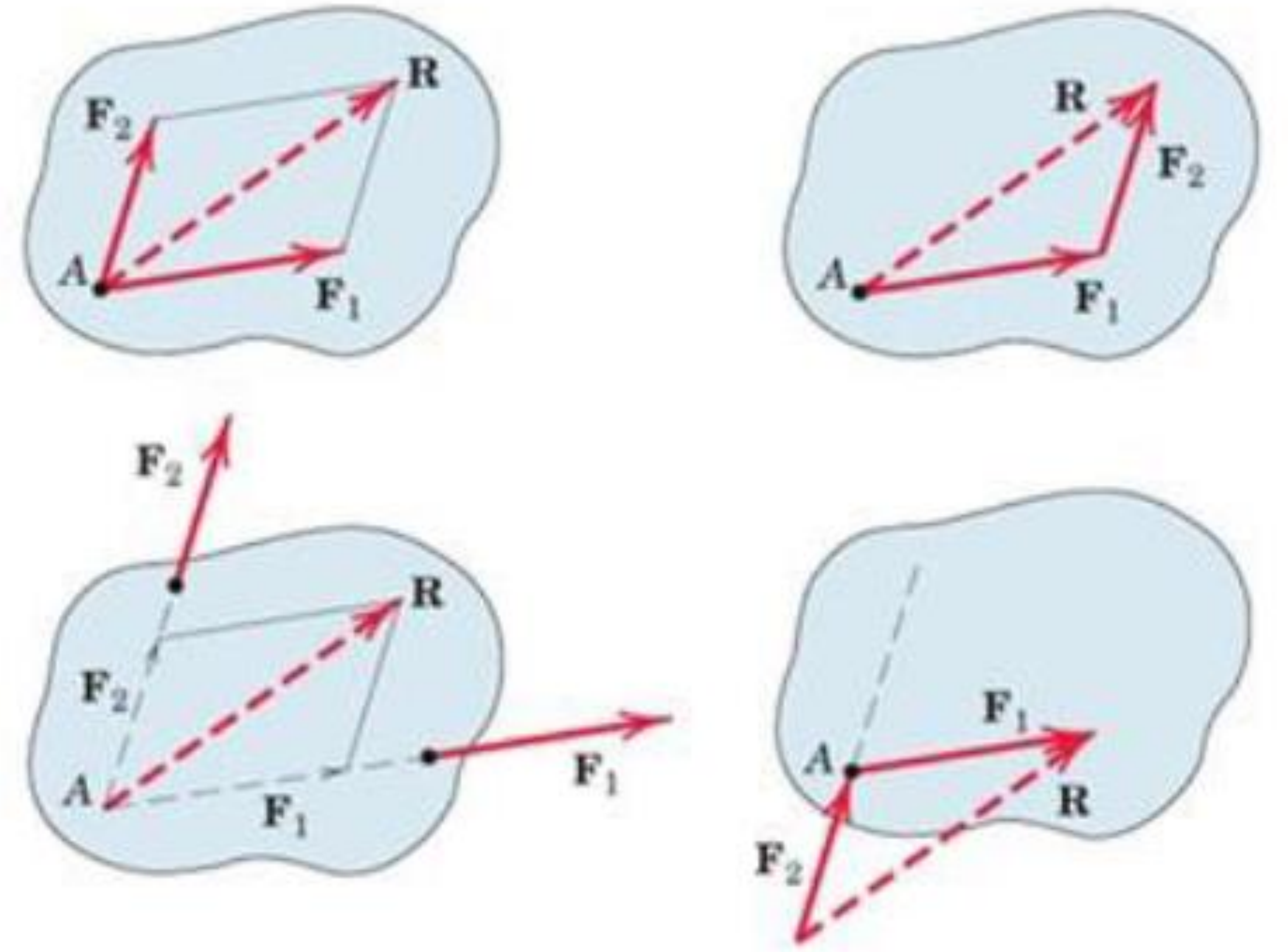
Principle of Transmissibility

- In Statics & Dynamics, we are not interested in the deformation of the body (internal effect)
- For such cases, the line of action of the force is more important than the point of application

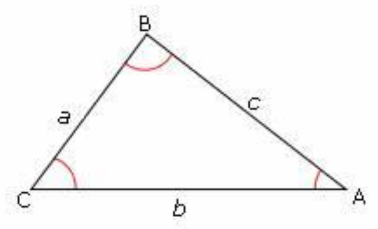


Force **P** will have the same effect whether applied at point *B* or *A*

Principle of Transmissibility



Use the parallelogram law to add/subtract vectors and assure that the line of action is preserved



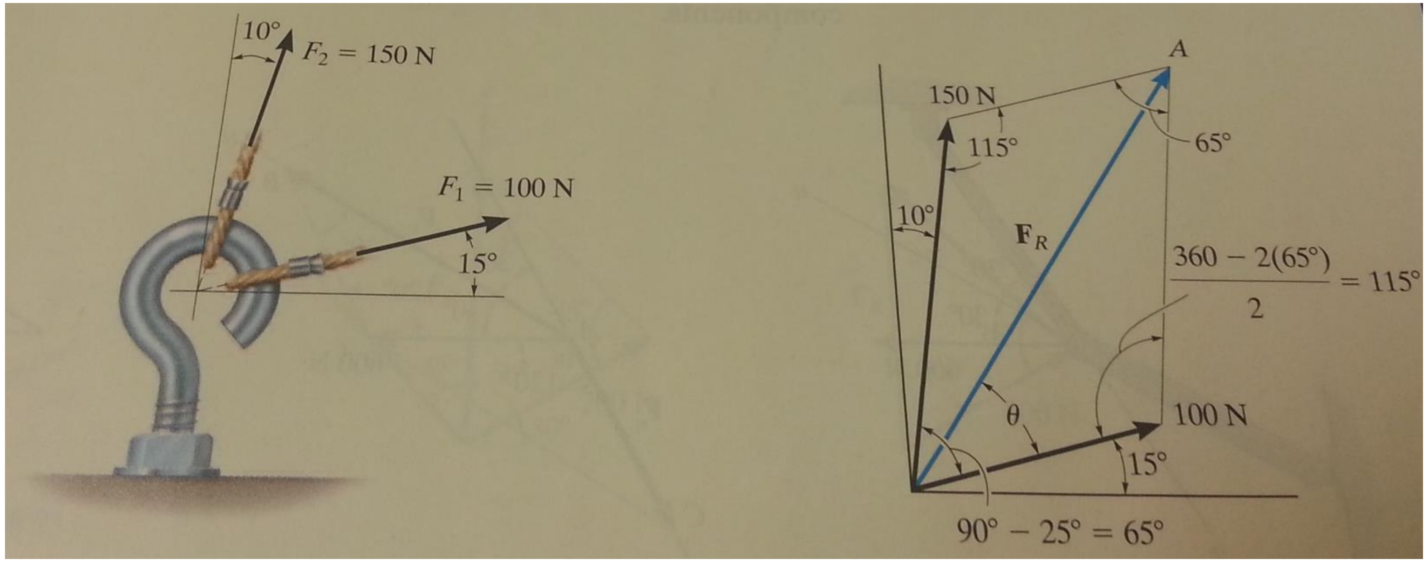
$$c^2 = a^2 + b^2 - 2ab \cdot cosC$$

$$b^2 = a^2 + c^2 - 2ac \cdot cosB$$

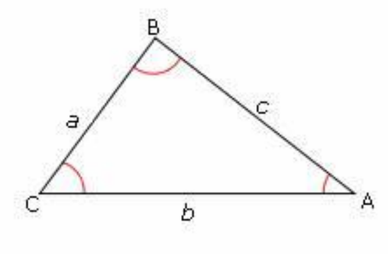
$$a^2 = b^2 + c^2 - 2bc \cdot cosA$$

Example

The screw eye in Fig. is subjected to two forces F1 and F2. Determine the magnitude and direction of the resultant force.



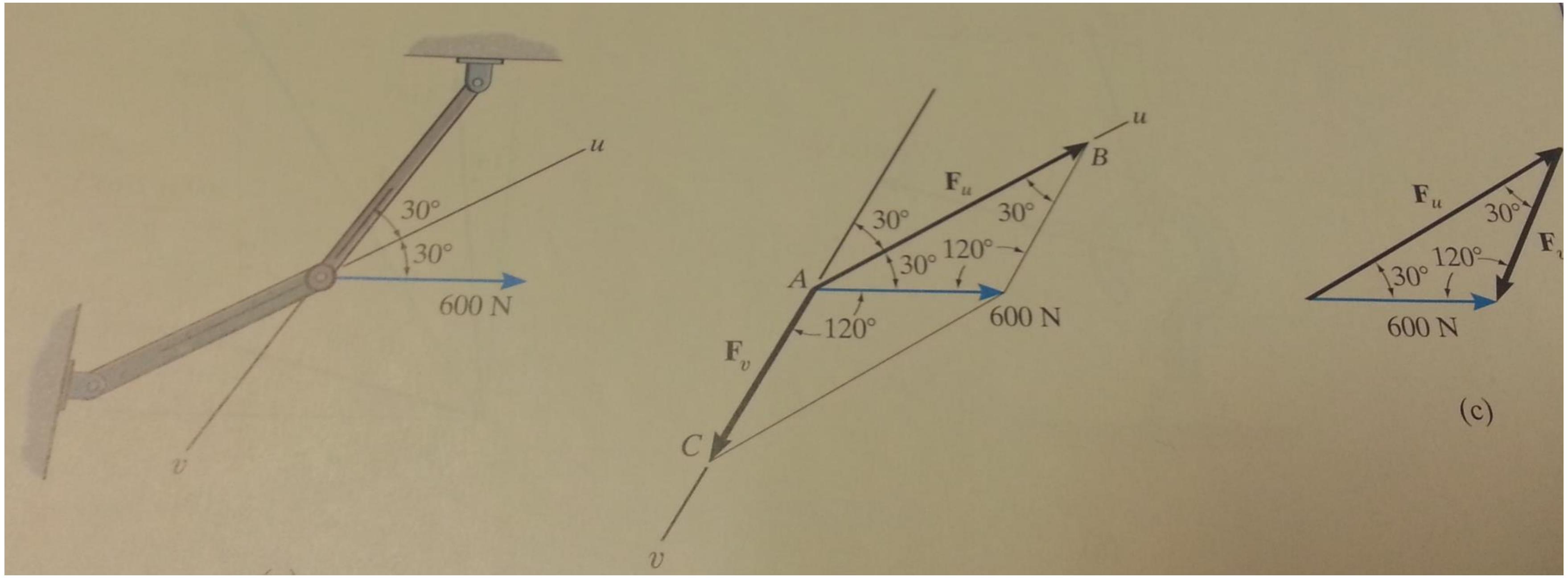
F_R =

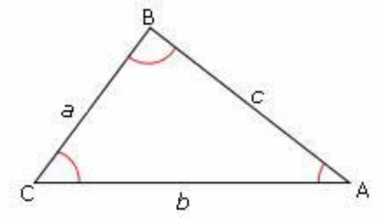


$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Example

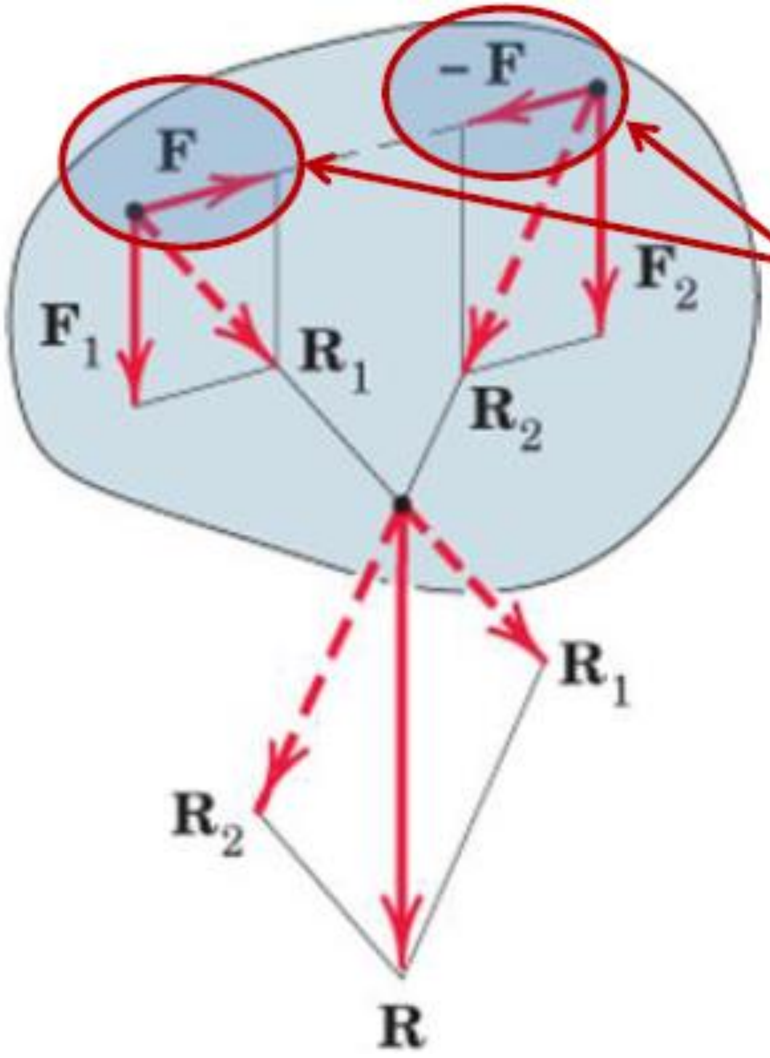
Resolve the horizontal 600 N force in the Fig. into components acting along the u and V axes and determine the magnitude of these components.





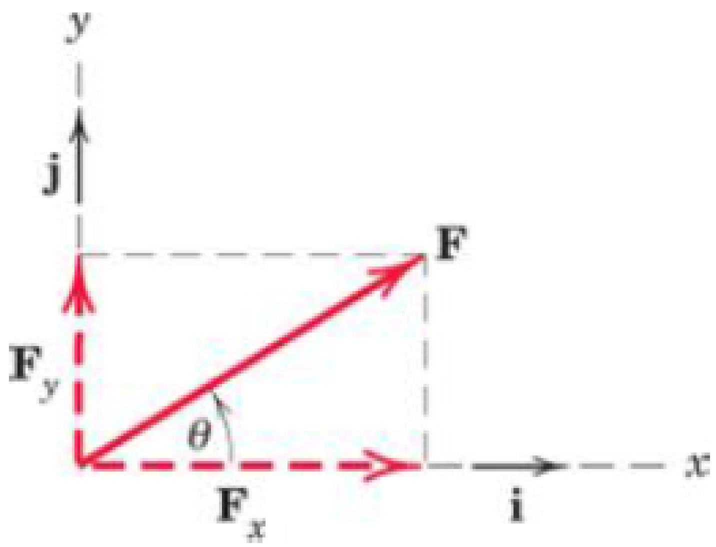
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Adding Parallel Forces



Adding two opposing forces **F** and −**F** with the same line of action will not alter the mechanics of the body

2-D Forces in Cartesian Coordinates



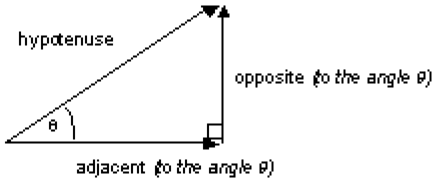
$$F_x = F\cos\theta, \quad F = \sqrt{F_x^2 + F_y^2}$$

$$F_y = F\sin\theta, \quad \theta = \tan^{-1}\ F_y / F_x$$

$$\vec{F} = \vec{F}_x + \vec{F}_y$$

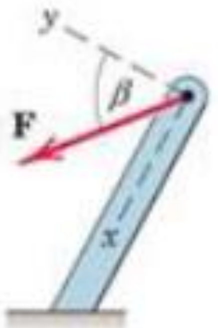
$$= F_x\hat{i} + F_y\hat{j}$$

Trigonometry

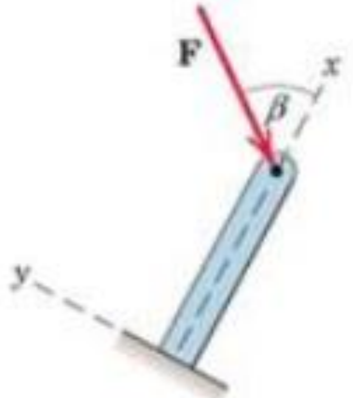


$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \cos\theta = \frac{adjacent}{hypotenuse} \qquad \tan\theta = \frac{opposite}{adjacent}$$

2-D Forces in Cartesian Coordinates

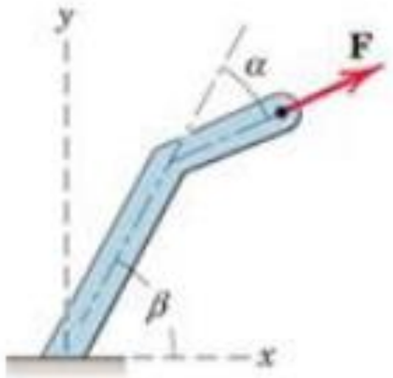


$$F_x = F \sin \beta$$
$$F_y = F \cos \beta$$



$$F_x = -F \cos \beta$$
$$F_y = -F \sin \beta$$

$$F_x = F \sin(\pi - \beta)$$
$$F_y = -F \cos(\pi - \beta)$$

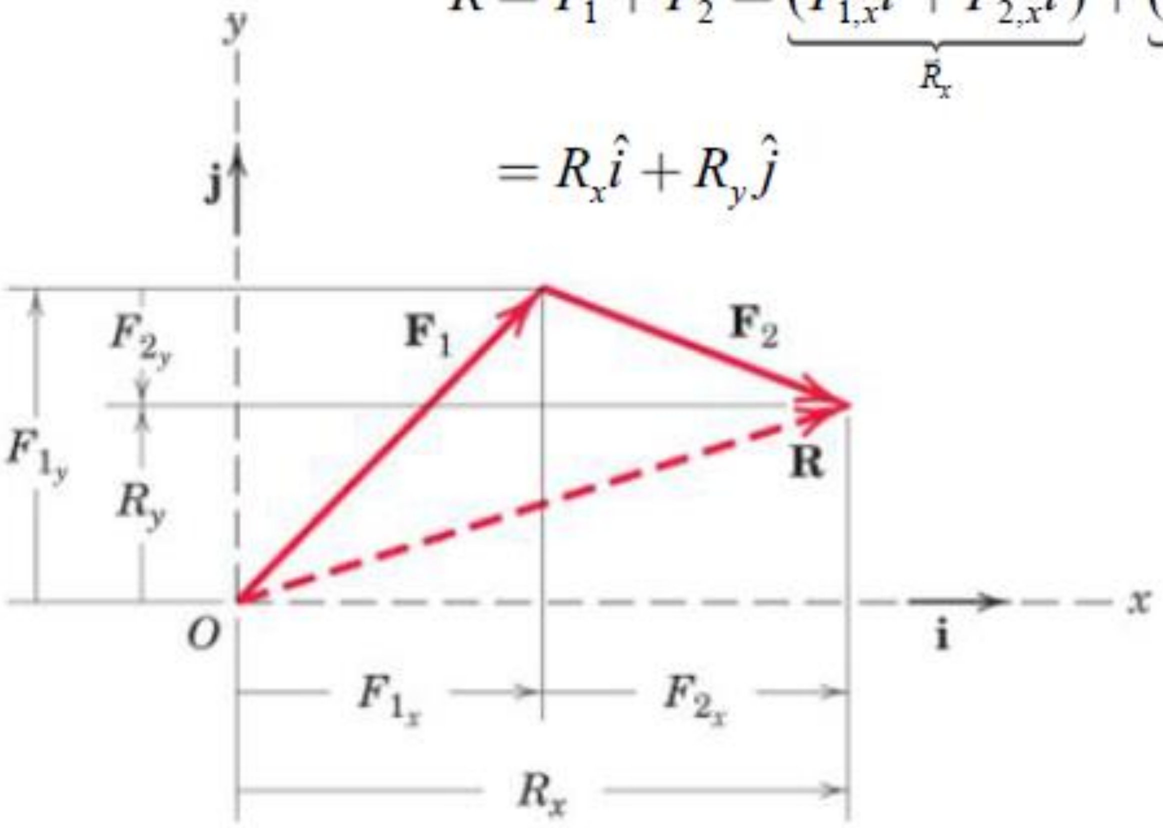


$$F_x = F \cos(\beta - \alpha)$$
$$F_y = F \sin(\beta - \alpha)$$

Summation of 2-D Forces

$$\vec{R} = \vec{F}_1 + \vec{F}_2 = \underbrace{(F_{1,x}\hat{i} + F_{2,x}\hat{i})}_{\vec{R}_x} + \underbrace{(F_{1,y}\hat{j} + F_{2,y}\hat{j})}_{\vec{R}_y}$$

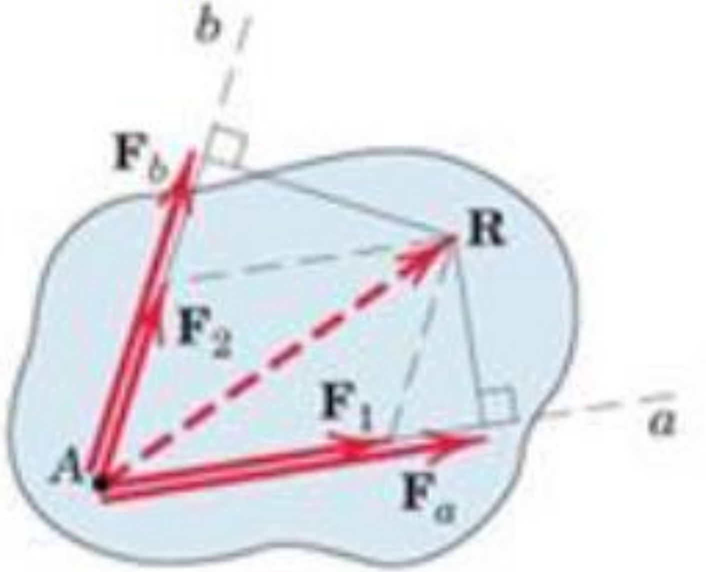
$$= R_x\hat{i} + R_y\hat{j}$$



Projection vs Components

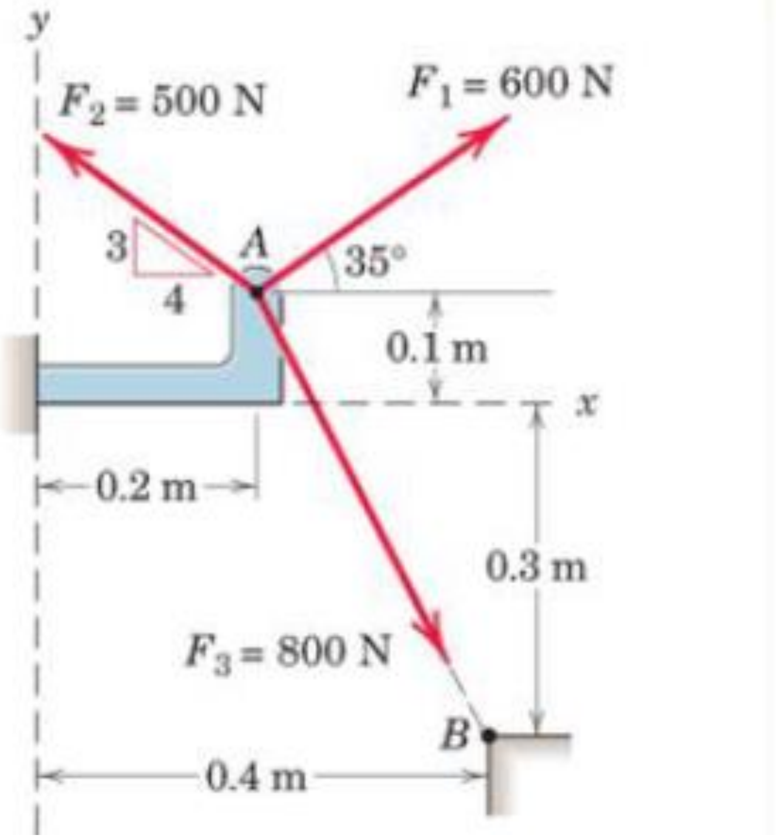
- Be careful not to confuse the components of a vector along two axes and the projection of the vector onto those axes
  - Components of $\mathbf{R}$ along a- & b-axes are $\mathbf{F}_1$ & $\mathbf{F}_2$
  - Projection of $\mathbf{R}$ onto a- & b-axes are $\mathbf{F}_a$ & $\mathbf{F}_b$

$$\vec{F}_1 \neq \vec{F}_a$$
$$\vec{F}_2 \neq \vec{F}_b$$



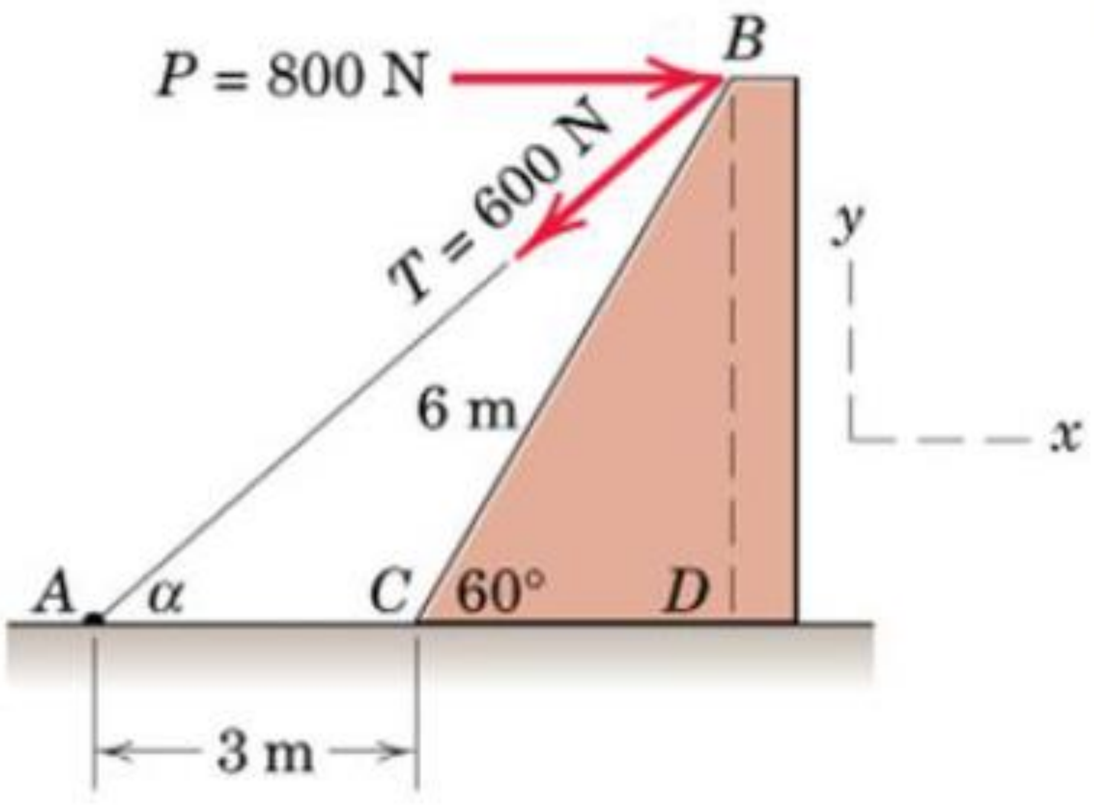
Example: 2-D Forces

- Determine the x- and y- components of the forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ acting at point A of the bracket shown below



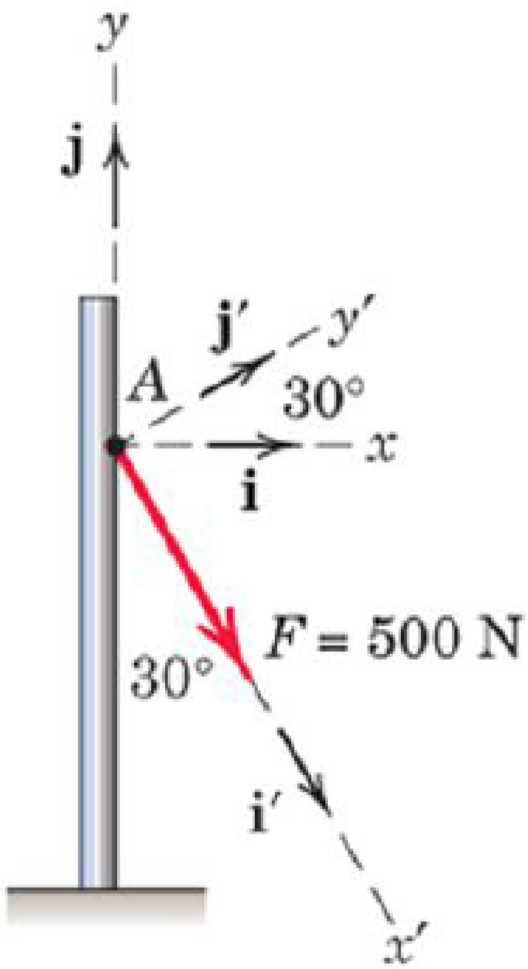
Example: 2-D Forces

- Determine the resultant due to the forces **P** and **T** shown below



Example: 2-D Forces

- For the 500 N force $\mathbf{F}$ shown below
  - Write $\mathbf{F}$ in terms of the unit vectors $\mathbf{i}$ and $\mathbf{j}$
  - Determine the scalar components of $\mathbf{F}$ along the x′- and y′-axes
  - Determine the scalar components of $\mathbf{F}$ along the x- and y′-axes



Example: 2-D Forces

- Determine the projection of the resultant $\mathbf{R}$ of the forces onto the b-axis